Midterm 1

Math 525: Probability

March 15, 2018

Last name, first name: ______________________________

Section number: ______________________________

User ID: ______________________________

Question:	1	2	3	Total
Points:	30	35	35	100
Score:				

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Question 1 (30 points)

Consider a dinner at a **circular table** with a total of $N \geq 4$ (distinct) guests including Barbie (B), Ken (K), and the famous probabilist Émile Borel (E). B, K, and E wish to sit together (i.e., there cannot be another guest between any two of them). How many possible arrangements satisfy this requirement?

Question 2 (35 points)

Let X and Y be **independent** real-valued random variables defined on some probability space. Let $f\colon \mathbb{R} \to \mathbb{R}$ and $g\colon \mathbb{R} \to \mathbb{R}$ be **monotone bijective** Borel measurable functions. Show that $f \circ X$ and $g \circ Y$ are also independent.

Hint: two random variables U and W are said to be independent if

$$\mathbb{P}(\{U \leq u\} \cap \{W \leq w\}) = \mathbb{P}\{U \leq u\}\mathbb{P}\{W \leq w\}$$

for all choices of $u, w \in \mathbb{R}$.

Question 3 (35 points)

Let $(X_n)_n$ be a sequence of random variables that are bounded from below by an integrable random variable Y (i.e., $X_n \geq Y$ a.s. for each n). Suppose that $\lim_n X_n = X$ a.s. Show that $\mathbb{E}[X] \leq \sup_n \mathbb{E}[X_n]$.

Hint: Fatou's lemma states that when $(W_n)_n$ is a sequence of nonnegative random variables, $\mathbb{E}[\liminf_n W_n] \leq \liminf_n \mathbb{E}[W_n]$.