

Math 525: Assignment 5

1. Let X , Y , and Z be square integrable random variables such that X and Y are independent. Show that

$$\mathbb{E}[XYZ] \leq \sqrt{\mathbb{E}[X^2] \mathbb{E}[Y^2] \mathbb{E}[Z^2]}.$$

2. Let X be an integrable random variable with moment generating function M . Show that $M(\theta) \geq e^{\theta \mathbb{E}[X]}$ whenever $e^{\theta X}$ is integrable.
3. Let p be a positive integer and $(X_n)_n$ be a sequence of random variables satisfying

$$\mathbb{E}[|X_n|^p] \leq f(n)$$

where f satisfies $\sum_n f(n) < \infty$. Show that

$$\lim_n X_n = 0 \text{ a.s.}$$

Hint: use Markov's inequality and the Borel-Cantelli lemma.