All of Statistics - Chapter 6 Solutions

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1.

Since $\mathbb{E}_{\lambda}[\hat{\lambda}] = \mathbb{E}_{\lambda}[X_1]$, the estimator is unbiased. Moreover, $\operatorname{se}(\hat{\lambda})^2 = \mathbb{V}_{\lambda}(X_1)/n = \lambda/n$. By the biasvariance decomposition, the MSE is equal to $\operatorname{se}(\hat{\lambda})^2$.

2.

If *y* is between 0 and θ ,

$$\mathbb{P}_ heta(\hat{ heta} \leq y) = \mathbb{P}_ heta(X_1 \leq y)^n = (y/ heta)^n.$$

Differentiating yields the PDF of $\hat{ heta}$ between 0 and heta as $y\mapsto n(y/ heta)^n/y$. Therefore,

$$\mathbb{E}_{ heta}[\hat{ heta}] = \int_{0}^{ heta} n(y/ heta)^n dy = heta n/(n+1).$$

It follows that the bias of this estimator is - heta/(n+1) Moreover,

$$\mathrm{se}(\hat{ heta})^2 = \int_0^ heta n y (y/ heta)^n dy - \mathbb{E}_ heta [\hat{ heta}]^2 = heta^2 n/(n+2) - \mathbb{E}_ heta [\hat{ heta}]^2.$$

By the bias-variance decomposition, the MSE is $heta^2 n/(n+2) - heta^2 (n^2-1)/(n+1)^2$. Remark. $\hat{ heta}(n+1)/n$ is an unbiased estimator.

3.

Since $\mathbb{E}_ heta[\hat{ heta}]=2\mathbb{E}_ heta[X_1]= heta$, the estimator is unbiased. Moreover,

$$\mathrm{se}(\hat{ heta})^2 = 4 \mathbb{V}_ heta(X_1)/n = heta^2/(3n).$$

By the bias-variance decomposition, the MSE is equal to $\operatorname{se}(\hat{\theta})^2$.